Concluding Panel, MCM meeting Yale, june 2009.

It is very sad that Guerino Mazzola could not chair this event as originally planned. However, the ideas leading to the setting up of this panel were so original and so deep, that we all felt it would be a pity not to maintain the event, even though we will miss G's leadership, energy and (possibly) clarifications.
The original project was an open discussion of a music and dance performance that was given in Minneapolis, on a project of Mazzola. This project was embodying a mathematical concept, or formula, called the Fourier transform, in the gestures of several dancers interacting with electronic music. Underlying this project is Mazzola's present research, on the theory of gestures.
In his absence we will begin with examples of Fourier gestures, in a very naïve and practical sense. This will allow a painless transition from discrete models to continuous models, which I think is the epistemologic rupture that we are presently experiencing (more or less unconsciously). Just recall all the papers you have witnessed during these last two days featuring such a transition (or revolution, as the case might be). From there, we will graduate to gestures, sketching in five minutes, top, Mazzola's theory of hypergestures. At this point we will be ready and willing to witness the performance of *Dancing the Violent Body of Sound* (on DVD). We will then leave it to Ian Quinn to connect the loose ends, and conclude this journey.

In the sound domain, Fourier theory is well known and understood, providing (in most cases) a decomposition of a complex sound as a sum of sine waves. More abstractly, it is an ismorphism between a real-life, physical universe, and the Fourier space (consider for signals the chronogram on the one hand et the spectrogram on the other). For sound, this stems from the structure of the cochlea, which is replete with resonators tuned to different frequencies. At this point, I would like to stress that one cannot hear music without a brain: there is an important reorganization of the data provided by the cochlea (including filtering, for instance).
As it is thus avered that the mind is capable of performing Fourier analysis, it is quite conceivable that this capability is used in other domains than sound. How nice if we could put forward domains of music — say rhythms, scales, chords — wherein the Fourier decomposition would provide directly meaningful information! After all, this is also called 'harmonic analysis'.
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Some of us are aware that such is indeed the case. After a shaky start, when readers of Lewin first paper in JMT threatened to resign their subscriptions, we had to wait several decades 'til Ian Quinn revived the concept in his now famous paper on prototypes and the lanscape of chords (with the exception of Dan Tudor Vuza's seminal paper on rhythmic canons, which stated clearly the importance of the Discrete Fourier transform of a set, and even enlarged this to continuous compact groups, a special case of which yields the continuous Hexa Thm that was presented yesterday). The number of Fourier fanatics has been growing steadily (you will have noticed a few around here, and we hope to outnumber the voice-leading zealots soon), and for good reason: even Dmitri Tymocsko had to confess that consideration of the magnitude of Fourier coefficients

yields 99% of the information available in voice-leading space, though it is not at all contrived to do so.
Fourier decomposition provides valuable characterizations of some special subsets of discrete universes, like ME sets, FLID, Z-related sets, intervallic relation-retrieval, which are extremely useful for composers, for instance; in a minute we will ask Moreno Andreatta and Carlos Agon, the OpenMusic visual environment software gurus, to show us how easy and informative it is to navigate in a Fourier landscape.
But after the Quinn earthquake and discussions of Fourier coefficients features at the John Clough memorial days in 2005, Thomas Noll and I took a good look at DFT of finite subsets of the continuous circle. Moving Fourier coefficients is strictly equivalent to moving the original, physical elements, as the Fourier transform is bijective, but it stresses interesting, abstract (and global) qualities:

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While I hold on to the microphone I would like to show a specific application of this modified DFT. I am extremely pround of it, because
1) whatever I do next, it will probably remain my most famous result ever
2) it connects me with Johann Sebastian Bach, no less
3) I discovered this by pure chance, and without any additional effort.
In other words it is the complete antithesis of my usual research.

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Now we will witness how appopriate software allows to make the most of the Fourier qualities. An aside here while we change computers: so many prominent features of scales/chords/rhythms are directly expressed by the size of a Fourier coefficient that we have launched in Ircam a program of cognitive experiments, testing whether there is direct perception of those. Production of artefacts with OpenMusic will be a vital tool in this program.

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May I introduce the world's first and only Fourier DJ, Dr Thomas Noll. He has been slyly preparing his change of career for a very long time, with a paper on 'Fourier scratching' that anticipated the actual implementation by some years. So far this Fourier scratching deals with two dimensions of sound, texture and loudness, within a fixed periodic rhythm.
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As we have seen, there seems to be a paradigm of continuous models that is gaining momentum: even without mentioning diverse continuous approaches in the papers presented these last few days, we remember that in his discrete typology of chords and their prototypes, Quinn repeatedly advocated a 'fuzzy' approach; see also orbifolds (Callender, Quinn, Tymocsko); and gestures (Mazzola).
So now is the time to talk a little about gestures, which transform movement into music (is this a general defnition for the ultimate musical instrument ?).
It might be fairly obvious to an instrumentist, that the gesture connecting two notes is not spanning a void, but loaded with meaning; still a theory of gestures is anything but trivial.
I remember being stuck during the composition of a tango. I was freed twice by gestures, gestures in the mind: once when I realized that the sound I wished for on a certain note required vibrato - so the piece is intended for clavichord, not for piano; and also when I visualized the dancing moves (a *boleo*, for the connoisseurs) which allowed the notes to flow onto the score.

Most of us will recall similar experiences with gestures inside the music. M.'s preoccupation with gestures comes from his being both a pianist (free-jazz improviser) and a mathematician and theorist, who had already introduced Lie operators and other mathematical monsters for the purpose of modelizing interpretation: the agogic for instance suggests continuous variations around the metronomical, lifeless, canonical rendering (think of `groove' functions for midi instruments), like the free-flowing movements of a limb, be it metaphorical or physical. Let us hear some deep-frozen Rachmaninov and agree upon the existence of a missing (continuous) dimension, not unlike those eleventh small dimensions that string physicists want us to believe in: ...

Moreover, these ideas became instrumental (no pun intended) in helping M. resolve a major personal crisis (may 18, 2002) that he describes in 'La vérité du beau dans la musique':
« at that time [having just published the huge [ToM]] I was thinking that this framework would be sufficient for every and all music-theory problem that we might encounter and that we could solve them within this framework.
But [...] I realized that my practice as a jazz musician was making use of different strategies that the one described in [ToM] ...»

So M. realized that a whole level of meaning was missing in his theory; a level that was moreover essentially pertaining to his practice as pianist. He had to build a whole new theory that would articulate gestures with the existing framework.
(As M. himself points out, this kind of perspective reversal, from the mind to the corporeal intelligence, is not unheard in cognitive sciences, or A.I. for instance.)

A midi recording (or a wax roll...), a musical score, are deep-frozen gesture. So is composition: writing takes into account the possible gestures of the instrumentist, or even the dancer. Interpretation, among other things, can be seen as de-frosting, thawing,

bringing the gesture back alive and in interaction with (say) musical instruments, thus producing the sound/music.
The idea of musical scores as frozen musical gestures dates back (at least) to Adorno. The main problem of course is recreating the gesture from its frozen instance (this is not limited to interpretation as we will see). (the inverse problem is not easy, either as we have heard in ‘s conference). As Jean Cavaillès wrote,
« Comprendre, c'est attraper le geste et pouvoir continuer ».
To understand is to get the gesture and be able to continue.
All this fits in perfectly with David's Lewin approach of the same problem,
Lewin in *Generalized Music Intervals*: " if I am at *s* and wish to get to *t*, what *characteristic gesture* should I perform in order to arrive there ?"

The natural way for a formalization of this is the theory of diagrams (for laymen, things interspersed with arrows). This was perfectly natural for Mazzola, whose *magnum opus* addresses Juliet's question 'what's in a name' with the theory of denotators - the answer is 'an arrow'.
But this is only the skeleton (eg, points *s, t*). With, say, Lewin GIS, we are one step further, with the concept of a transformation between *s, t* (eg a rotation).
The huge remaining problem is the vacuity of arrows: a rotation (say a 3x3 matrix) is not continuously moving point *s* to point *t*, it is still — it is STILL (adjective). Just a snapshot of the movement, not the movement itself. What Lewin really asked is: what's in an arrow ?

Possibly it is the experience of jazz, with its higher degree of freedom, that allowed Mazzola to take the next *Giant Steps*, leading *Miles* away from these fairly simple concepts.
Mazzola describes (hyper)-gestures as (homotopy) classes of continuous paths joining the heads and tails of the arrows. This is getting very complicated, but only because Algebraic Topology is meant to give rigorous definitions to things like 'any movement from *s* to *t*". An astounding result, published in JMM, is that such gestures correspond to mathematical formulas (eg equations between symbols). Recalling the opinion of many artists (not only musicians) and also David Lewin that gestures manifest the attitude of 'someone inside the music, an idealized dancer/singer', this leads us to the notion of mathematical expressions pushing their way inside the music.

Fortunately there is no need to go into details today. Let us get down to earth with one of the simplest expressions. The interval between two pitch-classes *s, t* is a rotation, i.e. in complex notation, multiplication by $\exp(2i\pi\,(t-s)/12)$.
This is the snapshot, the arrow. Turning it into a gesture is best made by introducing the map $u \to \exp(2i\pi\,(t-s)u/12)$, $0 \le u \le 1$.
Now these maps $u \to \exp(2ki\pi u/12)$ form the basis of Fourier decompositions, which hence appear as one the simplest examples of gestures.

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Now we can understand a little better the dance and music performance that we are going to witness. Prefigured in Mazzola's *Flow, Gestures and Spaces in Free Jazz*, the idea is to start from a Fourier series, here the seminal case of Fourier decomposition of a sound. It is embodied (the word is apt) by the movements of dancers, each impersonating a $c_k \exp(2i\pi kt/n)$. This rotating technique is inspired from East Java traditional dance practice, which Rachmi Diyah Larasati had to teach to the dancers to begin with. The jumping and walking movements, and even the different rotation techniques, are obviously in relation with the common indonesian martial art practice, as we will see quite well in some parts.
Captors on the dancers' wrists allow to turn these movements into music via a computer system: on each wrist, a flex sensor controls the sound volume of a partial, an accelerometer controls its frequency. Hence the dancers distort the harmonics of a cello recording by Schuyler Tsuda . During rehearsals, the dancers learned to use the sensors in order to fine-tune their movement's tempo. (The most obvious correlation is between the 'fundamental' dancers (in red dress) and the loudness of the bass sounds.)
Variety in the dance is obtained through the violent interventions of 'police dancers' — the two guys in saffron tunics, who execute a kind of `kata' (combination of martial moves). This idea is dear to the choregraph, Diyah Larasati, who also appears as a free agent during the dance. She is particularly interested in the relationship between violence, dance and embodiment in Indonesia. The cello composition with electronic distorsion via the Max software is the work of Schuyler Tsuda. Mathematician and musician William Messing collaborated with Mazzola on the mathematical aspects of the project. The professional recording on video is due to Dag Yngvesson. Toni Pierce-Sands was responsible for rehearsals.

Though the relationships between gestures, formulas, music have been proved as rigorous theorems, this project is meant to make more apparent the correspondence at least in this very special case. Let us now witness these Unidentified Fourier Objects in this music and dance and Fourier piece, entitled *Dancing the Violent Body of Sound*.

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Now please Ian, as the inviting power, tell us your opinion about all this.